A Note on Using the "Svensson procedure" to estimate the risk free rate in corporate valuation

By Sven Arnold, Alexander Lahmann and Bernhard Schwetzler

October 2011

1. The risk free interest rate in corporate valuation

The risk free interest rate is needed as an input factor to calculate the cost of equity of a firm. After the recommendation by the German institute of CPA´s (IDW) the so called “Svensson method” has gained significant attention as procedure to derive an estimate for the risk free rate. This note gives a short overview.

When estimating the cost of equity of a firm practitioners and researches alike face a technical problem whenever there is a non-flat yield curve: The CAPM as a one – period model asks for a single unique risk free interest rate whereas in practice the yields of riskless government bonds differ for different times to maturity.

2. Yield-curve, term structure and the Svensson procedure

Dealing with risk free interest rates in corporate valuation requires knowledge of all different interest rates over the entire range of times to maturity (T). The **yield curve** graphically presents the yields to maturity (YTM) of riskless government bonds. In contrast the **term structure** displays exclusively the yield of zero-bonds as the spot rates over the different maturity. When looking for appropriate discount rates the spot rates should be the first choice; YTM are derived by looking at all bonds with the same maturity. The problem is that there are very few zero-coupon bonds without any bankruptcy risk. Bonds issued by the German state, or any other bond with an AAA rating issued by a state, are almost free of bankruptcy risk but these bonds are no zero-coupon bonds. This problem can be solved by using a “bootstrapping” procedure to derive the spot rates out of a set of coupon bonds and their market prices. An additional problem is that even when considering all government bonds their payments will not cover the entire range of the maturity spectrum. When estimating the term structure the point is to fill up the “gaps” in the times to maturity.

In an attempt to solve both problems Svensson (1992, 1994) extended the earlier results by Nelson and Siegel (1987) and developed a functional relation to estimate the term structure. The functional relation by Svensson is given by

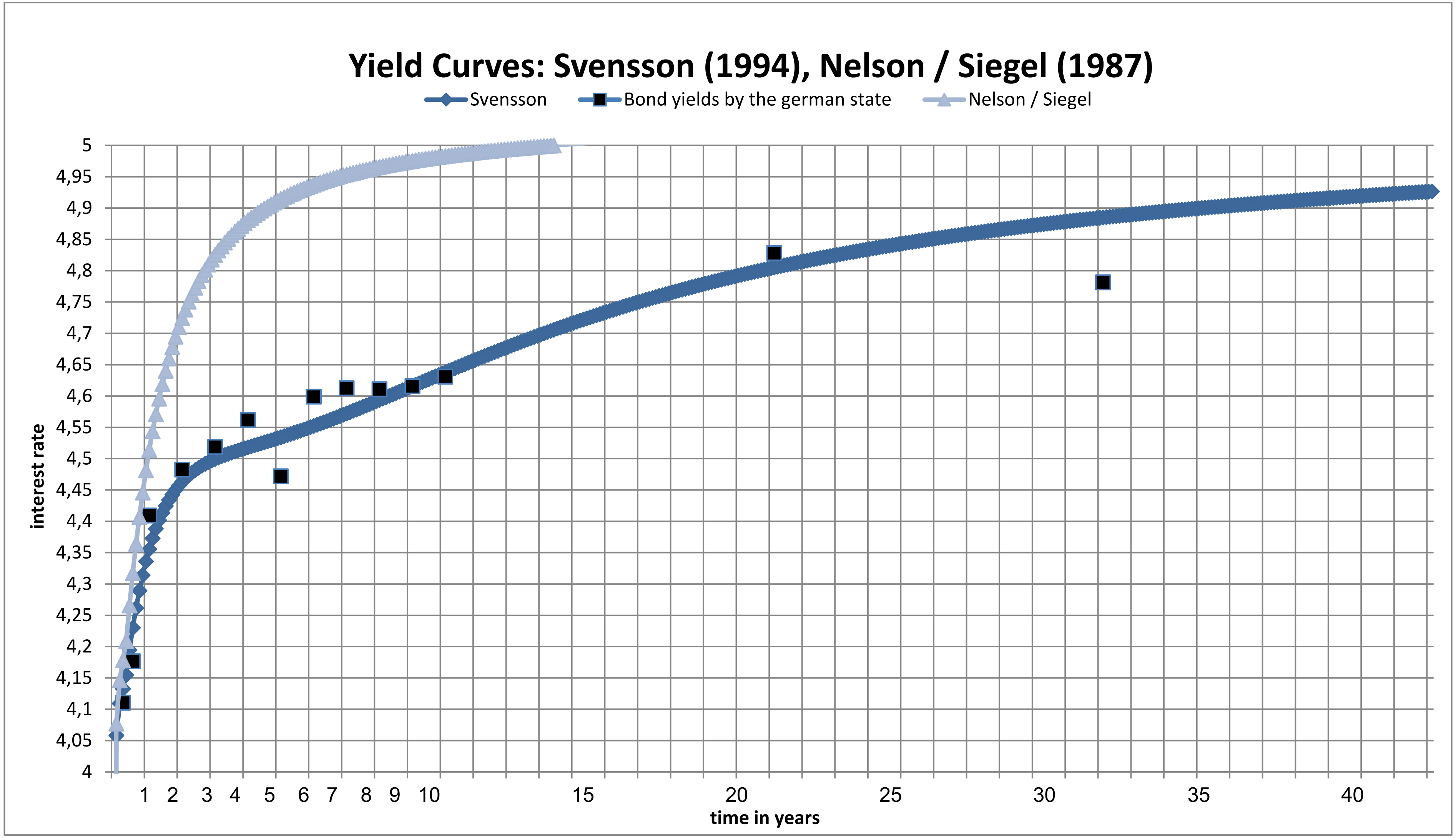
$$r_f(T,\beta) = \beta_0 + \beta_1\left(\frac{1-e^{\frac{-T}{\tau_1}}}{\frac{T}{\tau_1}}\right) + \beta_2\left(\frac{1-e^{\frac{-T}{\tau_1}}}{\frac{T}{\tau_1}} - e^{\frac{-T}{\tau_1}}\right) + \beta_3\left(\frac{1-e^{\frac{-T}{\tau_2}}}{\frac{T}{\tau_2}} - e^{\frac{-T}{\tau_2}}\right), \qquad (0.1)$$

with $r_f(T,\beta)$ being the risk free spot rate over time to maturity T as a function of the Beta-factors. β_0 to β_3 and τ_1,τ_2 are computed by a non-linear optimization program aiming to minimize the squared deviation between estimated and true interest rates. The functional form of equation (0.1) allows for a wide range of potential shapes of the term structure not covered by simple linear or log-based estimation procedures.

For German government bonds Deutsche Bundesbank regularly estimates and publishes the factors on its websites.

http://www.bundesbank.de/statistik/statistik_zeitreihen.php?lang=de&open=&func=list&tr=www_s300_it03c

In order to show potential differences between the estimation procedures proposed by Nelson and Siegel (1987) and by Svensson (1994) we've estimated the term structure for Germany at the 20th of June 2007. Graph (1) displays the results.



Yield Curves: Svensson (1994), Nelson / Siegel (1987)
Svensson
Bond yields by the german state
Nelson / Siegel
interest rate
5
4,95
4,9
4,85
4,8
4,75
4,7
4,65
4,6
4,55
4,5
4,45
4,4
4,35
4,3
4,25
4,2
4,15
4,1
4,05
4
1 2 3 4 5 6 7 8 9 10 15 20 25 30 35 40
time in years

The Nelson-Siegel procedure overestimates the spot rates for T higher than one year. The Svensson method is more precise and has the benefit to “smooth out” a potential market mispricing due to illiquidity as e.g. in the case of the 30 year Bundesanleihe.

Graph I shows that the assumption of a flat term structure with a unique interest rate of all maturities is not very realistic. In this case we observe a difference between the spot rate with maturity 3 months and the one with maturity 30 years of 0.8%. Ignoring this difference would have significant influence on the value of company.

Deutsche Bundesbank does not directly publish the term structure, but provides data and estimates as input for the calculation. Using the parameters published on the website, you can estimate the German term structure on your own.

From Q4 the term structure estimates for Germany based on the Bundesbank data will be calculated and provided by www.finexpert.info every quarter.

3. How to proceed from here

Knowing the term structure of riskless bond/asset does not yet completely solve any problem in corporate valuation. One still has to incorporate the different spot rates into the discounting procedure in the case of a non-flat term structure. The valuation literature proposes offers several ways to do this:

a) Directly applying the spot rates as discount rates

Directly using risk free spot rates as discount rates for present value calculations requires risk-adjusted cash flow figures as certainty equivalents. As the CAPM theoretically allows for calculating market based certainty equivalents, this procedure is not very common.

b) Deriving a single, equivalent risk free interest rate

The basic idea here is to find a unique single risk free rate that, if applied upon the cash flows yields the same present value than using the time to maturity – specific spot rates. The derived unique rate is then serving as the risk free interest rate in the CAPM approach. The result of this procedure depends on the time structure of the cash flows discounted. In the valuation literature you find recommendations of a perpetual or a constant growth timing pattern to be assumed. The equation to be solved is then

$$\sum_{t=1}^{\infty}\frac{S[CF](1+g)^t}{(1+r_t)^t}=\frac{S[CF](1+g)}{r-g}$$

Here r_t denotes the spot rates for corresponding maturities, r the unique risk free interest rate to be derived and g the growth rate assumed for the cash flows. As the risk free rate is directly applied as discount rate, the cash flows should already be risk adjusted; so S(CF) denotes the certainty equivalent of the cash flows to be valued. In practice however this risk adjustment is widely ignored when using this procedure.

An extension of this idea is to take the risk premium in the cost of equity into account when setting upon the equivalence equation:

$$\sum_{t=1}^{\infty}\frac{E[CF](1+g)^t}{(1+r_t+z)^t}=\frac{E[CF](1+g)}{r+z-g}$$

If one uses the (unadjusted) expected value of the cash flows risk is taken into account by the risk premium z. Both approaches yield slightly different results for the unique risk free rate.

c) Deriving forward rates and computing maturity specific cost of equity

Knowing the term structure allows to derive the one period forward rates over the entire range of periods available. As these forward rates can be locked in as interest rate in the current period without any further risk, they may well serve as risk free future interest for any period t. Adding a risk premium for equity (derived via the CAPM) on top, allows then for deriving a “chain” of one-period cost of equity over the entire maturity range. In this case, the valuation of the cash flow stream should rely on a “roll back” approach stepwise using the one-period cost of capital.

Note that in all the three cases the standard assumption is that the risk premium on top of the risk free rate is independent from maturity i.e. constant for all time to maturities.

A final problem left in a) and c) is coming from the fact that in many countries (including Germany) the time to maturity spectrum of bonds is limited to maximum of 20 to 30 years. Thus, for the calculation of the terminal value there is no equivalent interest rate of a bond with an infinite time to maturity. In this case science and practice alike recommend to use the yield of the bond with the longest time to maturity available as an estimate for the return of a bond with an infinite lifetime.

References

Nelson, C. R. und A. F. Siegel (1987), Parsimonous modeling of yield curves, Journal of Business, 60, 4, pp. 473 – 489.

Svensson, L. E.O. (1994), Estimating and interpreting forward interest rates: Sweden 1992 - 94, IWF Working Paper 114, September.

http://www.bundesbank.de/statistik/statistik_zeitreihen.php?lang=de&open=&func=list&tr=www_s300_it03c